Adiabatic muon cooling channel with Li lenses and high field solenoids

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Abstract

Muon cooling channel is considered including: Li lenses for final focusing and ionization cooling of muons; 30-50 T short solenoids and 4 T transport solenoids for adiabatic matching of the lenses; 100-200 MHz linacs for acceleration. Ionization energy loss of muons is about 100 MeV in each lens of length 1 m, at average beam momentum 190-250 MeV/c. It is shown that transverse emittance 60-70 μm and transmission about 80% are achievable in the channel including 9-10 lenses. It is shown that particles loss caused by fast growth of longitudinal emittance is the main constraining factor.

1 Introduction

Lithium lens is a perfect device for muon ionization cooling because it furnishes both ultimately strong transverse focusing and acceptably small rms angle of multiple Coulomb scattering accompanying the ionization energy loss [1]. Potentially, Li lens can afford beta-function of several mm which opens the way to reach the beam transverse emittance of several tens μm.

However, it is a challenging task to design a whole cooling channel combining several Li lenses and other required parts, first of all accelerating systems (linacs). The main problem is matching of parts with very different inherent beta-functions which are typically 1 cm or less in the lenses and about 1 m or slightly less in the linacs. Chromatic aberrations are extremely strong in similar conditions, especially having regard to a fast growth of energy spread of the beam in the cooling channel with reasonable low muon energy and without emittance exchange. It has been shown earlier that achievable transverse emittance increases almost in order of value from theoretical limit increasing up to 0.3-0.5 mm when moderate-field solenoids [2] or plasma lenses [3] are used for the matching.

Sufficiently smooth variation of focusing field might be a solution of the problem assuring adiabatic transition from low beta-function region to high one. However, rather long transition length could be required for the case to satisfy the adiabaticity conditions. Typically, it may be as much as several tens cm under above mentioned conditions. It is apparent also that the area should be free of condensed materials to avoid an extensive emittance growth due to particles scattering at relatively high or even at a middle beta-function. Therefore, the result cannot be reached by a graduate decrease of the lens gradient with help of an appropriate shaping of their ends.

However, recent progress in area of high temperature superconductivity [4] offers a possibility to realize this idea by insertion to the lattice relatively short solenoids with field 30-50 T. Having an inherent beta-function several cm, similar solenoids can be adiabatically matched with Li lenses using a transition region 10 cm or less which can be produced by bell-shaping ends of the lenses or even by using of their fringe field [5]. Similar cooling channel is investigated in this paper including: 10-12 Li lenses for ultimate focusing and cooling; attendant HF solenoids for adiabatic matching; transport solenoids for RF cavities or linacs. Several variants of the channel are considered with an average beam momentum 190-250 MeV/c, initial emittance 0.4-1 mm, and 100-200 MHz accelerating system.

2 Theory

An ideal Li lens of radius b_{Li} with current J_{Li} creates azimuthal magnetic field $B_\varphi = G_{Li} r$ with the gradient

$$G_{Li} = \frac{2J_{Li}}{cb_{Li}^2} \tag{1}$$

For a particle of momentum p, corresponding beta-function is

$$\hat{\beta}_{Li} = \sqrt{\frac{pc}{eG_{Li}}} \tag{2}$$

Ionization cooling of a beam produced by the lens provides a possibility to reach the equilibrium normalized rms emittance:

$$\varepsilon_{eq} = \frac{\hat{\beta}_{Li}}{2} \frac{d\overline{\theta^2}/dz}{d\gamma/dz} \beta^3\gamma^2 \tag{3}$$

with $d\overline{\theta^2}/dz$ as rms angle of multiple Coulomb scattering. Characteristic dependence of the parameters on normalized velocity β and energy γ are described by the expressions

$$\frac{d\overline{\theta^2}}{dz} \propto \frac{1}{\beta^4\gamma^2}, \qquad \frac{d\gamma}{dz} \propto \frac{1}{\beta^2} \tag{4}$$

(of a little importance logarithmic factors are neglected here). Taking into account these relations, one can rewrite Eq. (3) in the form:

$$\varepsilon_{eq} = S_{Li}\beta\hat{\beta}_{Li} \simeq 0.0085\,\beta\hat{\beta}_{Li} \tag{5}$$

with S - factor which depends on the material being 0.0085 for Li. Rms radius of such a beam is

$$a = \sqrt{\frac{\hat{\beta}_{Li}\varepsilon_{eq}}{\beta\gamma}} = \hat{\beta}_{Li}\sqrt{\frac{S_{Li}}{\gamma}} \tag{6}$$

The lithium rod should have several times more radius: $b_{Li} = \alpha a$ with $\alpha > 1$ as the safety factor. Therefore, with Eq (1)-(6) taken into account, following expression can be obtained for the lens current:

$$J_{min} = \frac{ec\alpha^2\beta S_{Li}}{2r_0} = 15\,\alpha^2\beta\ (\mathrm{kA}) \tag{7}$$

where $r_0 = e^2/mc^2$ is the particle classic radius. Actually it is a minimal value which is required to keep the equilibrium emittance described by Eq. (5) (it is why the subindex "min" is applied). Other minimal parameters of the lens can be presented as functions of desirable equilibrium emittance:

$$G_{min} = 0.0026\,\frac{\beta^3\gamma}{\varepsilon_{eq}^2}, \qquad b_{min} = 11\,\frac{\alpha\epsilon_{eq}}{\beta\sqrt{\gamma}}, \qquad B_{min} = 0.028\,\frac{\alpha\beta^2\sqrt{\gamma}}{\varepsilon_{eq}} \tag{8}$$

(units: cm, T and T/cm). Beta-function provided by the lens is $\hat{\beta} = 120\,\varepsilon_{eq}/\beta$.

The case $p = 250$ MeV/c, $\alpha = 4$ is considered below as an example. According Eq. (7), the lens minimal current should be 220 kA in this case to reach equilibrium emittance regardless of its value. Other parameters of the lens are plotted in Fig. 1 against the equilibrium emittance.

Of course, actual capacity of the lens should be more than this minimal value to provide a possibility to decrease emittance at the cooling. With an actual beam emittance ε_{ac}, the cooling rate is

$$\frac{1}{\varepsilon}\frac{d\varepsilon}{dz} = -\frac{1}{p}\frac{dp}{dz}\Big(1 - \frac{\varepsilon_{eq}}{\varepsilon_{ac}}\Big) \tag{9}$$

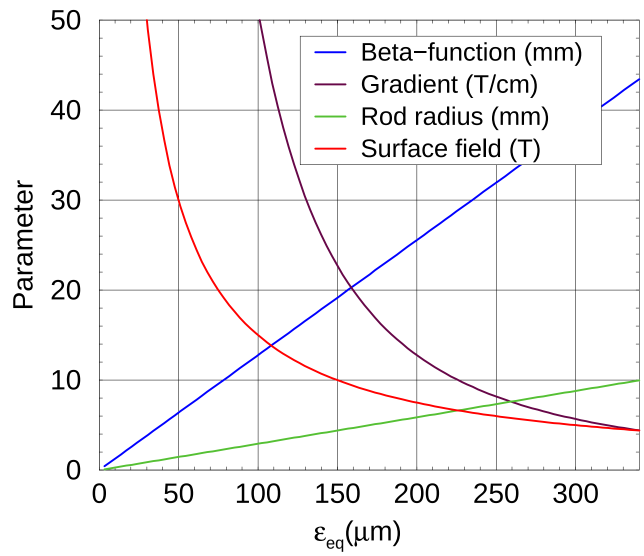

Figure 1: Li lens parameters against the equilibrium transverse emittance at beam momentum 250 MeV/c, and the lens current 220 kA (safety factor 4).

Correspondingly, the lens current, rod radius, and surface field should be more than it follows from Eq. (7)-(8), to accept sufficiently larger beam. Probably, surface field B_{Li} is the most constraining factor under the circumstances. Then achievable (equilibrium) and acceptable emittances of the lens are

$$\varepsilon_{eq} = 0.0505\sqrt{\frac{\gamma\beta^3 b_{Li}}{B_{Li}}}, \qquad \varepsilon_{ac} = \frac{0.17}{\alpha^2}\sqrt{\gamma\beta b_{Li}^3 B_{Li}} \tag{10}$$

(units: cm, Tesla). Both of them are plotted in Fig. 2 against the lens radius at the beam momentum 250 MeV/c, safety factor $\alpha = 4$, and the lens surface field $B_{Li} = 10$ T or 20 T. Crossing of the identically colored lines marks the equilibrium point with parameters:

$$b_{Li}\ (\mathrm{cm}) = 4.4/B_{Li}\ (\mathrm{T}), \quad J_{Li} = 220\ \mathrm{kA}, \quad \varepsilon\ (\mathrm{cm}) = 0.15/B_{Li}\ (\mathrm{T}).$$

Consequently, at least 20 T Li lens is required to reach the beam emittance 75 μm at beam momentum 250 MeV/c. However, lenses with smaller field (but with larger radius) can be used in earlier stages of the cooling, not caus-

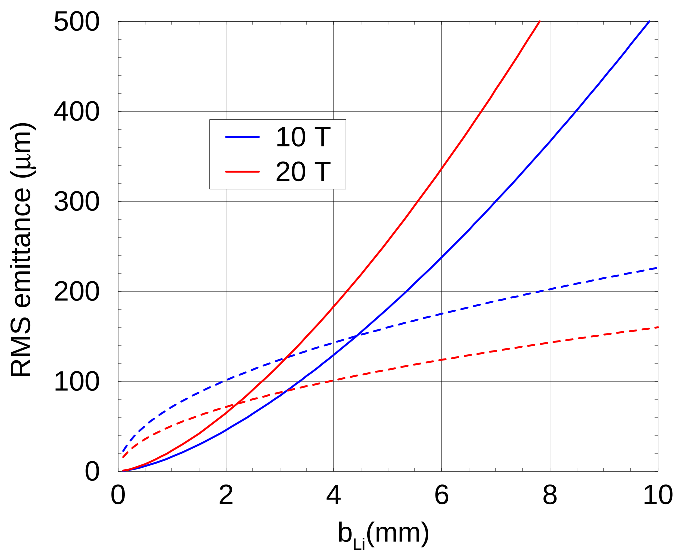

Figure 2: Beam emittances in a lens with surface field 10 or 20 T against the lens radius, at $p = 250$ MeV/c. Solid or dashed lines – acceptable or achievable (equilibrium) beam emittances.

ing an excessive degradations of the cooling rate, as it follows from Eq. (9). For example, with actual beam emittance 0.4 mm, the lens 10 T / 0.86 cm can provide the cooling efficiency 63% which is slightly less of the efficiency 81% which would be achieved with more strained lens 20 T / 0.68 cm.

As it has been pointed out in the Introduction, high field solenoids have to be placed near each Li lens to match them with high-beta parts of the channel. Thus the solenoid field B_S can penetrate into the lens changing the beta-function which is determined now by the expressions:

$$\frac{1}{\beta} = \sqrt{\frac{1}{\beta_{Li}^2} + \frac{1}{\beta_S^2}} \qquad \beta_S = \frac{2pc}{eB_S} \tag{11}$$

Formally, the condition $|d\beta/dz| \ll 1$ is needed for adiabaticity. Actually, the inequality

$$\left|\frac{d\beta}{dz}\right| < 0.25 \tag{12}$$

will be used further as a practical adiabaticity condition which assures against unacceptable emittance growth in going from Li lens to solenoid, or conversely (it will be confirmed by simulation in next section). Both the solenoid and the lens fields should be rather smooth functions of z for this. Practically, the solenoids in the consideration automatically satisfy the condition

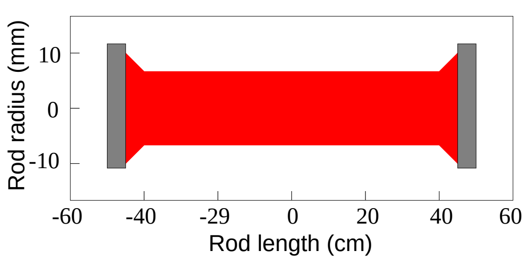

Figure 3: Schematic of Li rod for adiabatic matching. The rod radius is less of 1 cm in the center whereas the bell ends are 7-10 cm long. Gray zones present fringe field regions which require a special consideration.

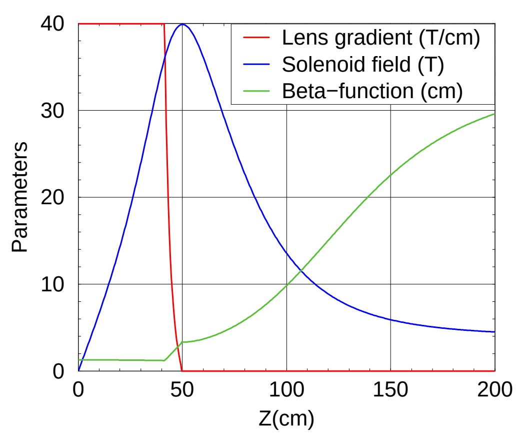

Figure 4: Solenoid field, Li lens gradient, and beta-function vs longitudinal coordinate.

through relatively large transverse size of their coils. However, sufficiently slow decay of Li lens gradient is accessible only with specially shaped edges, as it is schematically shown in Fig. 3. Inevitable increase of average beta-function within the lens can be minimized if the lens is long, and solenoid field is sufficiently large in its vicinity. An example of matching is presented in Fig. 4 where the lens gradient, solenoid field and total beta-function of a particle with momentum 200 MeV/c are plotted against the longitudinal coordinate. The beta-function growths from 1.3 cm to 3.3 cm in the end-bell of length 8 cm, which means increase of average beta-function by 13% if the lens is 1 m long. It is pertinent to note for comparison that almost doubling or average beta-function would occur in attempting to satisfy the same adiabaticity criterion when maximal solenoid field is reduced to 20 T.

It should be emphasized that the solenoid field presented in this graph is an odd function about the lens center. The field flip is an essential factor for full value transverse cooling, and it does not lead to unwanted impact on the focusing being performed inside the lens.

3 Cooling simulations

Several versions of the cooling channel will be considered in this section having a goal to compare and optimize their characteristics. However, all of them have some general features which are described in advance.

Each proposed channel consists of 4-5 similar in appearance cells one of them is being schematically shown in Fig. 5 in concert with axial magnetic field. The solenoid coils include a high-field part and a low-field (transport) one both of them being marked by blue in the picture. Geometry of the high-field part is sketched in Fig. 6 (it is not an engineering consideration because it is applied actually to make sure that used magnetic field is Maxwellian). The solenoid cells are geometrically identical but may bear different current in accordance with designed field, which never exceeds 50 T. The field is reversed in center of each Li lens (red) which circumstance does not violates adiabaticity and almost does not affect transverse focusing, because the solenoid contribution is small in the lenses except the end-bell parts. The lenses of length 1 m are applied in all versions, each providing ionization energy loss 90-100 MeV for central particle of the bunch, dependent on average beam momentum. Other parameters of the lenses vary with each channel as

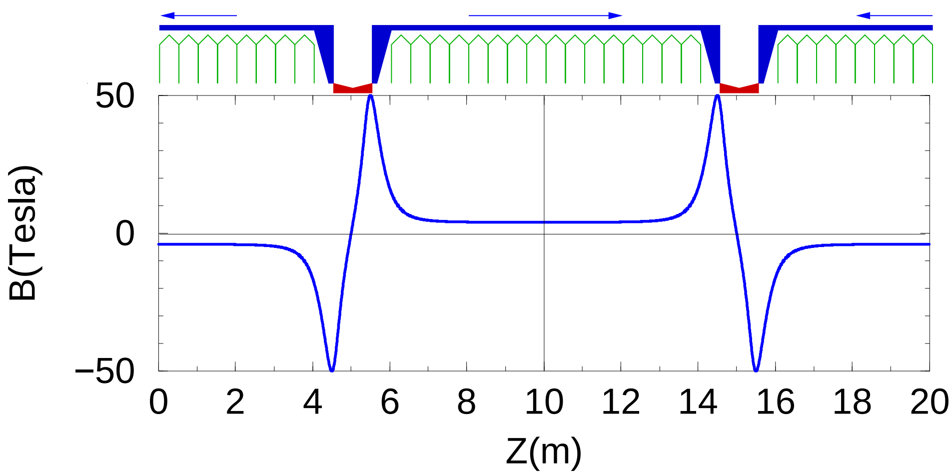

Figure 5: Fragment of the cooling channel. Red - Li lenses, blue - solenoid coils, green - RF cavities. Axial magnetic field is sown as well at the bottom.

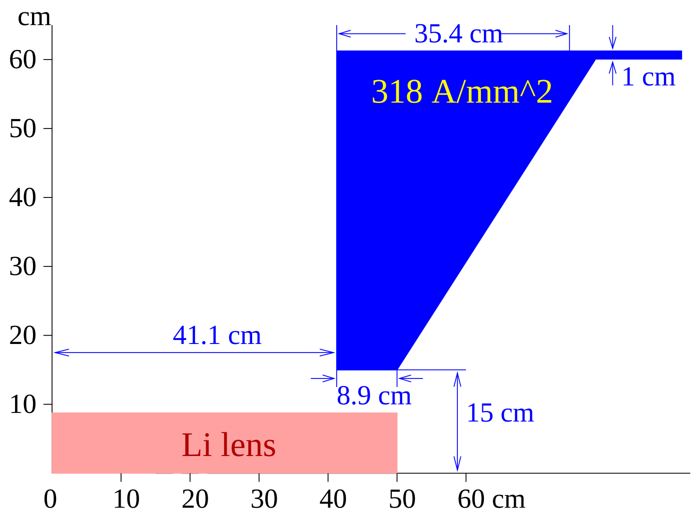

Figure 6: Configuration of the cell central part including Li lens (red) and high field coil. All the lenses differ in diameter and current, and the solenoid current density can differ in the high field part of the coil.

well as within the confines of any channel; however, surface field of the lenses does not exceeds 20 T. RF cavities or linacs 200 MHz / 16 MV/m have been considered in a base version, and possibility to apply lower RF is also investigated. It is assumed that the cavities are located inside of the transport solenoids of inner radius 60 cm, as it is sketched in Fig. 5 by green. Average beam momentum is 190-250 MeV/c depending on version. However, variation of energy about the average value is rather high in all versions having a range 90-100 MeV. Gaussian bunch with 4000 particles has been used for the simulations.

3.1 200 MHz channel for 250 MeV/c beam

The channel of length 120 m with 12 Li lenses for a beam with average momentum 250 MeV/c is considered in this subsection. The main characteristics of the channel and the beam are presented in Table 1. All solenoid cells have the same parameters in the case providing the field up to 50 T in the high field part and 4 T in the transport part (see Figs. 5 and 6). However, the lenses have dissimilar parameters which are listed in Table 2 and plotted in Fig. 7. Beta-functions in the lenses and near them are plotted in Fig. 8 for the beam momentum 250 MeV/c and the safety factor $\alpha = 3.75$. Actually, central beam momentum is rather variable being 300 MeV/c before any lens and about 200 MeV/c after it. Linear accelerators of frequency 200 MHz are placed between the lenses to compensate the energy loss. Each of them provides acceleration rate 11 MeV/m with synchronous phase 45^o. It is assumed that the linacs are placed inside the transport solenoids.

Evolution of the beam parameters is presented in Fig. 9. By the graphs, transverse normalized rms emittance decreases from 400 μm in the beginning of the channel to 85 μm in the end, whereas longitudinal rms emittance increases by factor about 10 from 1 mm to $\sim$ 1 cm. Particles loss comprises 16% being caused largely by decay and transverse aperture restrictions (7%+7%). Evolution of transverse and longitudinal phase space is also shown in Fig. 9 whereas their initial and final forms are presented in more detail in Fig. 10 and Fig. 11. While longitudinal motion gives a small contribution to the particles loss (about 2%), there is a significant distortion of longitudinal phase space, so the beam has very non-Gaussian distribution and actually reaches a verge of stability at the end of the channel.

Table 1. Cooler parameters for average beam momentum 250 MeV/c

The channel length	120 m
Number of Li lenses	12
Length of the lattice cell	10 m
Li lens length	1 m
Li lens gradients	34 - 95 T/cm
Linacs	11×8 m + 2×4 m, 200 MHz
The linac accelerating gradient	15.7 MV/m, 125 MV/linac
Reference particle energy rate	11.1 MeV/m, 89 MeV/linac
Synchronous phase	45^o
Initial transverse emittance	0.4 mm
Initial longitudinal emittance	1 mm

Table 2. Li lenses parameters ($\alpha = 3.75$).
Radius R(cm), Gradient G(T/cm), Surface field B(T), Current J(kA)

#	1	2	3	4	5	6	7	8	9	10	11	12
R	.53	.45	.39	.35	.32	.29	.27	.25	.24	.23	.22	.21
G	34	37	40	44	48	52	57	63	69	77	85	95
B	18	17	16	15	15	15	15	16	17	18	19	20
J	470	370	305	270	240	220	210	205	200	190	200	205

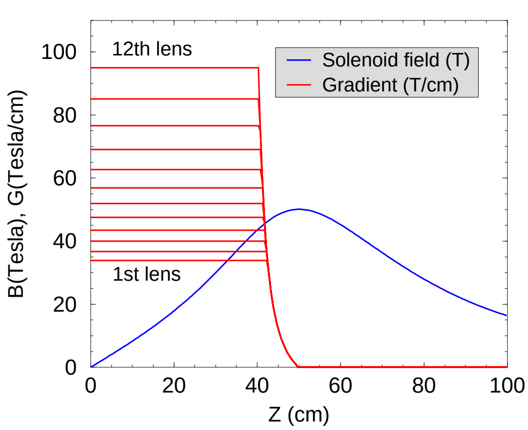

Figure 7: The solenoid field (blue) and the lenses gradient near the cell center.

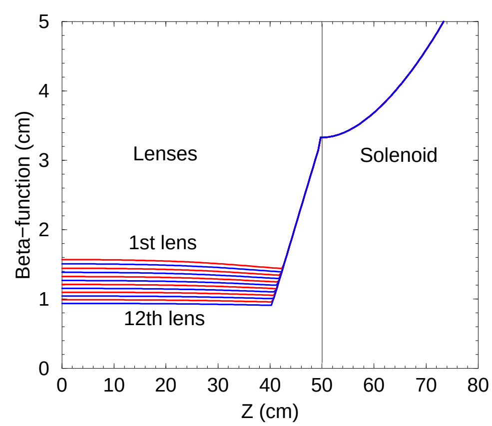

Figure 8: Beta-function near the cell centers decreases from 1.6 cm (first lens) to 0.9 cm (last lens).

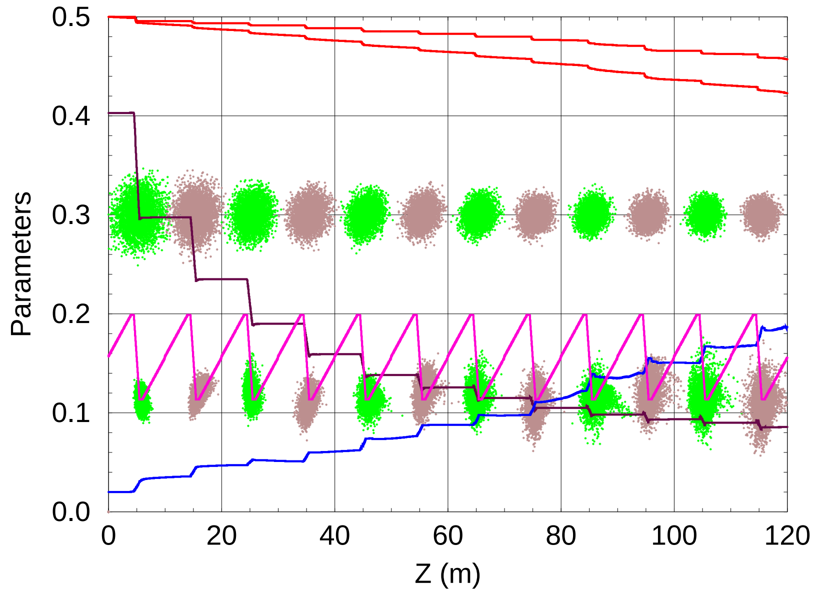

Figure 9: The beam parameters vs longitudinal coordinate. Red – transmission/2 without and with decay. Maroon – rms transverse emittance (mm). Magenta – normalized kinetic energy of reference particle/10. Blue – rms longitudinal emittance × 2 (cm). Phase space evolution is shown as well: top – transverse, bottom – longitudinal (green and brown, arbitrary units).

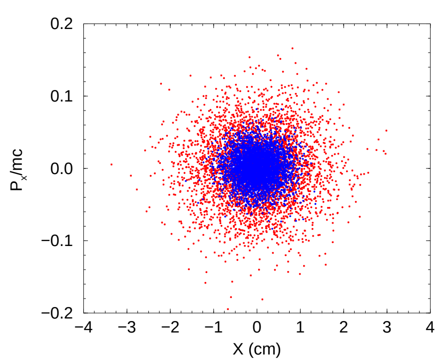

Figure 10: Transverse phase space: red - initial, blue - final

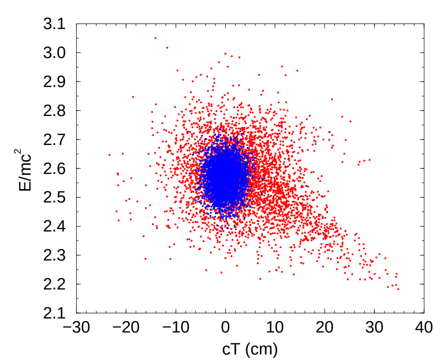

Figure 11: Transverse phase space: blue - initial, red - final

3.2 100 MHz channel for lower beam momentum

Lower beam momentum is very desirable to reach higher cooling rate and less transverse emittance. It could be expected to be especially effective in last stages of the cooling channel which largely determine the achievable beam emittance. To study such a possibility, the case is considered in this subsection with beam average momentum which gradually decreases from 242 MeV/c to 183 MeV/c lengthwise of the the cooling channel.

However, longitudinal motion is significantly deteriorated at lower momentum because of faster growth of the beam energy spread and diminution of the acceptable spread. Decrease of accelerating frequency is the most apparent way to compensate there effects, thus 100 MHz accelerating system is considered in the subsection. Accelerating gradient 11.1 MV/m is applied with synchronous phase 41^o-45^o, so that the particles energy rate in the channel does not exceed 7.9 MeV/m at this frequency, However, Li lenses of length 1 m are applied as before, so that longer linacs are required to compensate the energy loss. Therefore the cell length is increased up to 14 m, and total channel length is taken as long as 140 m (10 cells), because transmission falls abruptly after that. Maximal solenoid field increases along the channel as well being 30 T at the beginning and 50 T at the end. Other parameters are listed in Tables 3 and 4.

Table 3. Cooler parameters for beam momentum 240–190 MeV/c

The channel length	140 m
Number of Li lenses	10
Length of the lattice cell	14 m
Li lens length	1 m
Li lens gradients	13 - 96 T/cm
Linacs	9×13 m + 2×6.5 m, 100 MHz
The linac accelerating gradient	11.1 MV/m, 144 MV/linac
Reference particle energy rate	7.3-7.9 MeV/m, 95-102 MeV/linac
Synchronous phase	41^o-45^o
Initial transverse emittance	0.4 mm
Initial longitudinal emittance	1 mm

Table 4. Solenoid and Li lenses parameters of the lower momentum channel

N	1	2	3	4	5	6	7	8	9	10
B_S(T)	30	35	40	45	50	50	50	50	50	50
R_{Li}(cm)	1.00	0.76	0.59	0.44	0.37	0.30	0.27	0.25	0.23	0.21
G_{Li}(T/cm)	12.6	20.4	30.7	44.0	56.1	65.9	74.6	82.4	89.0	96.4
B_{Li}(T)	12.6	15.5	18.1	19.4	19.8	19.8	20.0	20.0	20.0	20.0

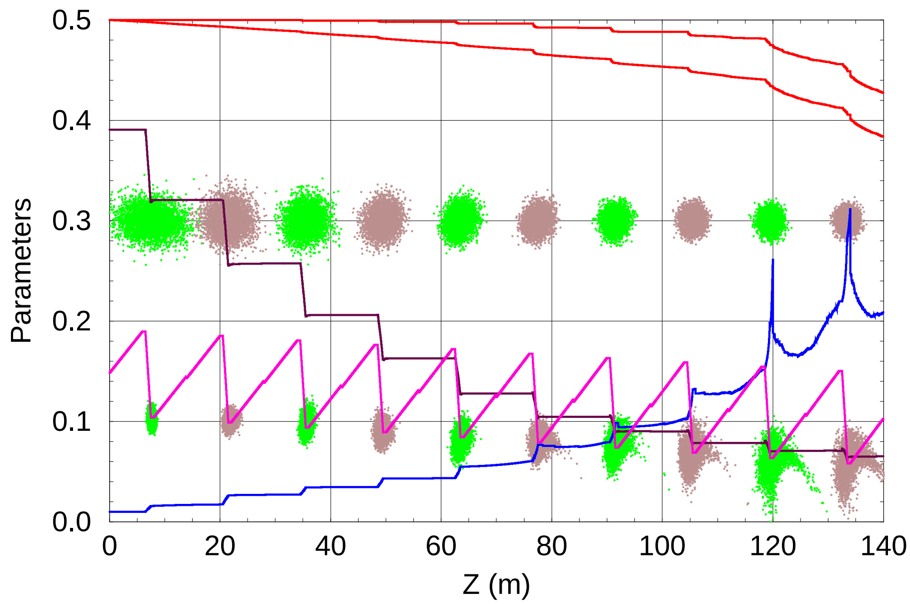

Figure 12: The beam parameters vs longitudinal coordinate. Red – transmission/2 without and with decay. Maroon – rms transverse emittance (mm). Magenta – normalized kinetic energy of reference particle/10. Blue – rms longitudinal emittance (cm). Phase space evolution is shown as well: top – transverse, bottom – longitudinal (arbitrary units).

Results of the simulation which are presented in Fig. 12 demonstrate a vague advantage of this system in comparison with previous one (Fig. 9). It is seen that transverse emittance / transmission achieve 71 μm / 82% after 9 cells (126 m), and 64 μm / 77% after 10 cells. However, the transmission falls crucially after 11th cell if it is added. Besides, it is pertinent to note about twice as fast growth of the bunch length and longitudinal emittance.

3.3 Cooling channel of larger acceptance

It is seen from Table 4 that surface field of lenses in the beginning of the cooler is significantly less as compared with the field of several last-named lenses. Therefore their radius can be increased with no excess of 20 T surface field limit. In particular, radius of the first lens can be enlarged almost by factor 1.6 which would allow to increase the channel acceptance by factor 2.5 and to cool a beam with initial emittance up to 1 mm.

Such modification of previous version is considered in this subsection. The main parameters of the channel are the same as in Table 3, except the initial beam emittances which are now: 1 mm transversely and 2 mm longitudinally. As before, the channel includes 10 Li lenses with adjacent high-field solenoids; however, their parameters are changed to afford more acceptance with the same adiabaticity conditions. Surface field of each lens is 20 T, and the solenoid field does not exceed 50 T. More detail information concerning the lenses and solenoids is presented in Table 5.

Evolution of the beam parameters at the cooling is shown in Fig. 13. Achievable beam parameters are somewhat worse in this case: transverse emittance is $73\,\mu$m and transmission is 74%, instead of $64\,\mu$m and 77% in previous version. The main constraining factor is growth of longitudinal emittance which rms value exceeds 30 mm in 10th lens resulting in a full stopping of many particles inside the rod.

Table 5. Li lenses and solenoids parameters
Surface field of each lens is 20 T

N	1	2	3	4	5	6	7	8	9	10
B_S (T)	30	32.5	35	37.5	40	42.5	45	47.5	50	50
R_{Li} (cm)	1.59	1.13	0.86	0.67	0.54	0.40	0.33	0.28	0.24	0.21
G_{Li}(T/cm)	12.6	17.7	23.2	30.0	38.0	49.8	60.1	71.9	84.5	96.4

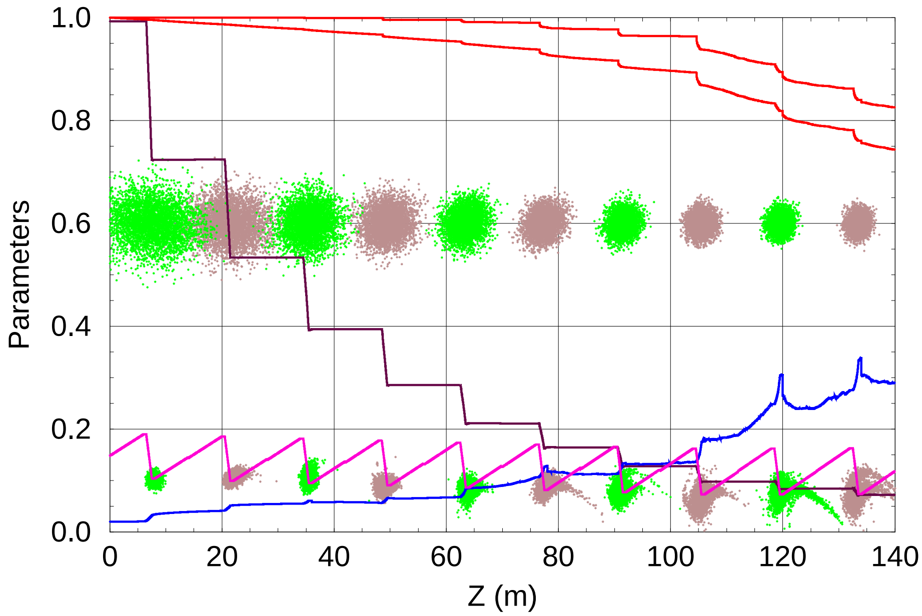

Figure 13: The beam parameters vs longitudinal coordinate at higher initial emittance. Red – transmission without and with decay. Maroon – rms transverse emittance (mm). Magenta – normalized kinetic energy of reference particle/10. Blue – rms longitudinal emittance (cm). Phase space evolution is shown as well: top – transverse, bottom – longitudinal (arbitrary units).

4 Conclusion

It is shown that short high-field solenoids can be used for adiabatic matching of Li lenses providing an opportunity to reach beta-functions which are inherent in the lenses. With 1 m long lenses and 100-200 MHz accelerating systems, transverse emittance of muon beam 60-70 μm is achievable in the cooling channel of length 120-140 m at average beam momentum 190-250 MeV/c. Growth of longitudinal emittance at the cooling without emittance exchange is the main constraining factor. Further lowering of accelerating frequency and, correspondingly, accelerating gradient would be required for all the lower beam momentum, resulting in unacceptably long cooling channel and decay loss.

5 Acknowledgments

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